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TACTICAL ROUTING FOR AIR TRANSPORTATION IN HKIA TERMINAL MANEUVERING AREA

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ABSTRACT

Terminal maneuvering area is a bottleneck for arrival air traffic flow, causing delays in peak times with excessive demands. To reduce the air traffic control's workload and mitigate congestion, numerous studies attempt to derive practical solutions from ground transportation and logistics to coordinate aircraft. Besides real-time practical infeasibility, the generic nature of proposed coordination models makes them directly inapplicable to realistic problems. Hence, we construct a simplified yet applicable model, deploying the mixed-integer linear programming approach, to obtain optimal coordination for arrival aircraft in the terminal maneuvering area of Hong Kong International Airport. In an effort to practicalize our model, we tune its parameters by applying extracted air traffic operation features from historical flight trajectory data. Results illustrate that our methodology is beneficial in tactically routing aircraft and suggesting optimal landing sequences in comparison with historical practices.

Keywords: Mixed-integer linear programming (MILP), arrival sequencing problem, Hong Kong International Airport (HKIA)

1. INTRODUCTION

Owing to the natural complexity inside the terminal maneuvering area (TMA), it is crucial to implement efficient decisions on aircraft scheduling operations and management. To tackle this issue, the aviation industry and community seek solutions from mathematical modeling. Due to the amalgamated nature of decision variables and operational constraints, mixed-integer linear programming (MILP) is regarded as an appropriate tool for air traffic management (ATM) research. In the past two decades, researchers frequently applied MILP for ATM research on several aspects. Richards and How (2002) planned flight trajectories concerning collision avoidance with the aid of MILP. The same year, Pallottino et al. (2002) solved two-dimension conflict resolution problems using MILP. In addition to these problems, the aircraft arrival sequencing problem (ASP) is another critical topic researchers aim to solve with MILP. A mixed-integer zero–one programming was used to solve static aircraft landing problems for single runway and multiple runways (Beasley, 2000). Besides the common static scenario, Samà et al. (2014) applied MILP to find the optimized aircraft landing sequence during disturbances. MILP could achieve a 93% average improvement in overall aircraft delay compared with the first-in, first-out (FIFO) strategy. More recently, MILP has been applied for problems with more realistic assumptions. The simultaneous optimization method for trajectory and sequence (SOM-TS) was developed with flight dynamics consideration and used MILP to solve the core time versus flight (TvF) cost function (Toratani, 2015, 2019). MILP was also implemented for route assignment with trajectory coordination for various types of vehicles (Matin-Moghaddam, 2020).

The previous study has provided an applicable and affordable scheme for the aviation industry. Yet, there are still limitations. First, due to the location-dependent feature of ATM study, the assumption required by MILP sometimes neglects critical components for aircraft arrivals in HKIA. For instance, typical arrival delay resolution strategies for air traffic controllers (ATCOs) are often ignored in the MILP formulation, including airborne holding (Toratani, 2015, 2019), vectoring (Samà, 2014; Toratani, 2015, 2019), and shortcut maneuvers (Beasley, 2000; Samà, 2014). Second, a specialized arrival sequencing modeling study at Hong Kong International Airport (HKIA) is missing. HKIA, one of the busiest airports worldwide, is located in the north of the Hong Kong airspace, which is not a standard airspace design.

The combination of high traffic demand and unconventional airspace infrastructure makes such aircraft arrival sequencing study at HKIA valuable. Although the COVID-19 outbreak has dramatically reduced the air traffic volume in Hong Kong, the recent recovery of the industry and the upcoming implementation of the third runway ensure the indispensability of this research. Furthermore, the interaction analysis between a low-fidelity optimization model (e.g., MILP) and a high-fidelity real-time operation (e.g., simulation) is often lacking. Without this consideration, some previous MILP models could not be compared with historical data. In this study, we present an MILP study at HKIA concerning the typical ATCOs arrival delay resolution strategies (holding, vectoring) and the interaction with a real-time simulation. The application for MILP in the tactical phase, i.e., the air traffic flow management (ATFM) stage to manage real-time situations, for aircraft arrival movements is presented. Section 2 describes the problem formulation of our mixed-integer flight coordination programming, while Section 3 offers the practical situation of arrivals in HKIA and the associated validation of our framework. Section 4 establishes the simulation results of two hours arrival flights and the comparison between actual and optimized arrival transit time inside the TMA. Lastly, the summary and future extension of this work are discussed in Section 5.

2. FLIGHT COORDINATION MODELING

2.1 Parameters and decision variables

We coordinate a set of flights, F, in a directed network $G = (W, S)$ where W and S are the set of waypoints and segments, respectively. Each flight $f \in F$ may originate from a particular point O^f ($O^f \in W$), yet it has only one common destination D ($D \in W$) since G represents the arrival network. Aircraft f has to find its way to the destination D through available segments, where the shortest path takes r^f time units. On each directed segment ij ($i, j \in W, ij \in S$) from waypoint i to waypoint j, we denote t_i^f as the time that flight f is at waypoint i and starts flying to the next waypoint j. It normally takes an aircraft s_{ij} time units, the standard flying time in the directed link ij, to reach j. However, the amount of time spent in one segment is subject to change according to the current situation of TMA. In the case of non-busy conditions, ATC tends to allow aircraft to fly faster or use short tracks, both of which are excluded in the model. On the other hand, delay strategies such as vectoring or even holding will be applied in overloaded circumstances. We use d_{ij} to denote the maximum vectoring time in link ij while n_i^f describes the number of holding patterns, each of h_i time units, that flight f has to proceed at waypoint i. To avoid any confusion caused by holding patterns, we stipulate that segment ij takes only holding patterns at i into account; holding at j is counted for the segment of j and its subsequent waypoint.

The binary decision variable x_{ij}^f is employed to capture the movement flow of flight f in segment ij. It is equal to one if and only if flight f passes ij. Besides, the algorithm also uses the previously mentioned t_i^f as another decision variable. The difference of t_i^f and t_j^f reflects short-track and vectoring decisions within the link ij. The number of holding patterns, on the other hand, needs to be specified by the integer decision variable n_i^f.

2.2 Objective

The most frequent objective function for such a traffic coordination problem is to seek minimum total traveling time. In the context of air traffic, we use the term "total transit time" to indicate the amount of time spent in TMA of all flights. Avoiding conflicts in the course of coordination may require flight f to deviate from its shortest path to its destination. In another word, instead of taking r^f time units to land, flight f needs a *de-facto* transit duration of $\left(t_D^f - t_{O^f}^f\right)$. Although our ultimate goal is to minimize the total transit time of all flights, we can break out the problem at individual flight levels by minimizing the actual transit time $\left(t_D^f - t_{O^f}^f\right)$ or the ratio of the actual transit time to the shortest time, $\frac{\left(t_D^f - t_{O^f}^f\right)}{r^f}$. Here, it is more natural to choose the ratio as a representation of the deviation of a flight, as different flights may have dissimilar shortest time periods to the airport due to different arrival routes. However, attempting to lower the above-mentioned ratio of one flight may increase those of other flights. Hence, it is more logical to find the optimal total transit time while maintaining the balance among flights in terms of deviation ratio. This idea can be expressed mathematically as follows:

$$\min(z), \tag{1}$$

subject to

$$w^f \frac{\left(t_D^f - t_{O^f}^f\right)}{r^f} < z, \tag{2}$$

where z is a real variable solely utilized for the optimization purpose and w is the weight representing the priority factor of a flight in the function of the number of passengers or the fuel consumption rate.

2.3 Constraints

Resource allocation. Constraint (3) enforces the flow balance at each node for every aircraft, analogous to the classic network flow problem (Ahuja *et al.*, 1988).

$$\sum_{j=1}^{N}\left(x_{ij}^f - x_{ji}^f\right) = \begin{cases} 1, & \text{if } i \in O^f \\ 0, & \text{if } i \notin \left\{O^f, D\right\}, \forall i \in N, f \in F. \\ -1, & \text{if } i = D \end{cases} \tag{3}$$

Travel time. When a flight f travels from waypoint i to waypoint j, it may use a shortcut or proceed delay strategies such as holding at node i and vectoring in middle. The following inequations describe the lower and upper bounds of spent time of flight f:

$$\begin{gathered} t_i^f + s_{ij}^f + n_i^f h_i \leq t_j^f + M\left(1 - x_{ij}^f\right), \\ t_i^f + s_{ij}^f + d_{ij} + n_i^f h_i \geq t_j^f - M\left(1 - x_{ij}^f\right), \end{gathered} \tag{4}$$

where M is a large positive number. M guarantees that the inequations become trivial when the flight f does not travel in link ij, otherwise its influence turns invalid.

Conflict. Aircraft are required to maintain a constant minimum separation in terms of geographical distance d with each other. However, to reduce the exponential influence of the problem's size, we apply conflict constraints only at waypoints. With this simplification, the model can be considered a macroscopic solution layer for ATC guidance, mainly focusing on the optimal sequencing of arrival flights. Conflicts at a microscopic level in each segment would be handled by other distributed methods such as reinforcement learning.

Besides, thanks to the convergence and deceleration characteristics of such an arrival traffic network, the velocities of different flights are approximately similar at the same waypoint. Let's consider two flights f and g flying in segment ij with velocities v_i at i and v_j at j. At the starting point of this segment, we care about the moments when the two aircraft exit holding patterns (if any).

$$\left|\left(t_j^f + n_i^f h_i\right) - \left(t_i^g + n_i^g h_i\right)\right| \geq \frac{d}{v_i}. \tag{5a}$$

Similarly, the same condition at j (without considering holding patterns) can be expressed as follows:

$$\left|t_j^f - t_j^g\right| \geq \frac{d}{v_j}. \tag{5b}$$

Wake separation. This distinguishing characteristic of air traffic to other modes of transportation illustrates the aerodynamic restriction behind an aircraft. Other aircraft are advised against flying in this dynamic restricted region, called wake separation, to avoid aerodynamic instability. The zone of influence depends positively on the aircraft's size, which differs from minimum separation.

$$\left|t_j^f - t_j^g\right| \geq \begin{cases} \dfrac{l^{fg}}{v_D}, & \text{if } t_j^f < t_j^g \\ \dfrac{l^{gf}}{v_D}, & \text{if } t_j^f > t_j^g \end{cases}, \tag{6}$$

where l^f is the wake separation requirement of aircraft f posing on the following.

Nature of decision variables. As mentioned previously, the real variables $t_i^f\,(i \in N, f \in F)$ determine the time plans of aircraft, in another word, the landing sequence. In addition, the employment of binary variables $x_{ij}^f(i,j \in N, f \in F)$ enforces the traffic flow while $n_i^f\,(i \in N, f \in F)$ are set as integer variables for counting the number of holding patterns.

$$\begin{aligned} &t_i^f \geq 0, && \forall f \in F, i \in N \\ &x_{ij}^f \in \{0,1\}, && \forall f \in F, (i,j) \in N \\ &n_i^f \in \mathbb{N}, && \forall f \in F, i \in N \end{aligned} \tag{7}$$

2.4 Extension of the network

Proposing traffic schedules at the tactical level demands the MILP program to be capable of dealing with the current scenario at any time. Flights may be inside or outside TMA by the time of computation. In the former case, we can simply create an additional (virtual) waypoint at the flight f's current location and assign it as the origin of f, O^f, and thus $t_{O^f}^f = 0$. In the latter situation, the origin waypoint will be the flight's entry with $t_{O^f}^f$ predicted by ATC.

3. APPLICATION TO HKIA TERMINAL MANEUVERING AREA

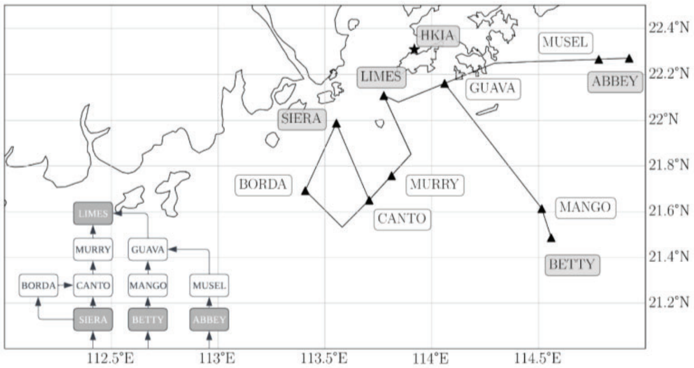

Figure 1. HKIA Standard Arrival Routes (STARs) leading to direction 07.
The simplified arrival network contains shown waypoints.

3.1 Study of HKIA arrival network structure

HKIA has two main arrival directions, 07 and 25, with the two corresponding Initial Approach Fixes (IAFs) including LIMES and TD. Choosing which IAF to approach depends on the current wind direction. That means, at one time, only one IAF is utilized. To avoid confusion, we subsequently opt for the IAF LIMES and arrival directions 07 for our study without losing the general complexity of the problem.

There are five arrival routes coming to LIMES, consisting of one from ABBEY, one from BETTY, one from CANTO, and the other two from SIERA, as illustrated with the solid lines in Fig. 1. Once an aircraft is assigned an arrival route, it is supposed to pass all waypoints of that arrival route. However, that flight might in fact ignore some waypoints while proceeding with shortcuts or delay strategies. In this study, we simplify the arrival network while maintaining the general structure of the arrival network. Besides the TMA entry waypoints (ABBEY, BETTY, CANTO, SIERA) and the selected IAF LIMES, we keep the waypoints where velocity or altitude requirements are imposed. The simplified network is shown in Fig. 1.

3.2 Determination of model parameters

Minimum separation, wake separation, and time duration of one holding pattern are specified in the Aeronautical Information Package (AIP) of HKIA[1]. Minimum separation d is set to four nautical miles (NM) while the standard time for one holding pattern h_i $(\forall i \in N)$ is set to six minutes, although this figure varies a lot in reality. We, however, keep this number constant in every situation, with a belief that shortcut and delay-related parameters, c_{ij} and d_{ij} $(\forall i, j \in N)$, are able to ensure the flexibility of the coordination solutions. Wake separation requirement, on the other hand, depends on the types of two adjacent aircraft as in the following table.

Table 1. Wake turbulence separation minima between two adjacent aircraft with their categories.

Preceding / Succeeding	Super (A380)	Heavy	Medium	Light
Super (A380)	-	5NM	7NM	8NM
Heavy	-	4NM	5NM	6NM
Medium	-	-	-	5NM
Light	-	-	-	-

- indicates wake separation is not applicable.
Instead, a safe minimum separation of 4 nautical miles (NM) should be applied.

Although short track and vectoring are common tools for ATC to obtain flexible plannings, their quantitative measurements are not formally mentioned in HKIA AIP. Besides, flying speeds at special waypoints do not usually align with requirements. It is therefore necessary to learn such statistics by wielding historical data. For instance, the two below figures show the flight distributions in terms of ground speed over the waypoint MUSEL and time spent in the segment MUSEL-GUAVA in May 2019. Both figures are broken down into two distributions corresponding to two levels of busyness within TMA, which are determined by the number of aircraft within TMA at that time. It is very likely that besides distance (through short track and vectoring), ATC also controls the aircraft's speed to mitigate congestion. The suggested model, on contrary, offers optimal time plannings as we apply the same speed profile for every airborne flight. After determining ground speeds by taking the mean value of historical aircraft speed, we simply compute the standard flying time of one segment by its length divided by its averaged ground speed. Meanwhile, the maximum vectoring time is defined as the difference between standard flying time and the 90th percentile value of the time distribution as illustrated in Fig. 2 (b).

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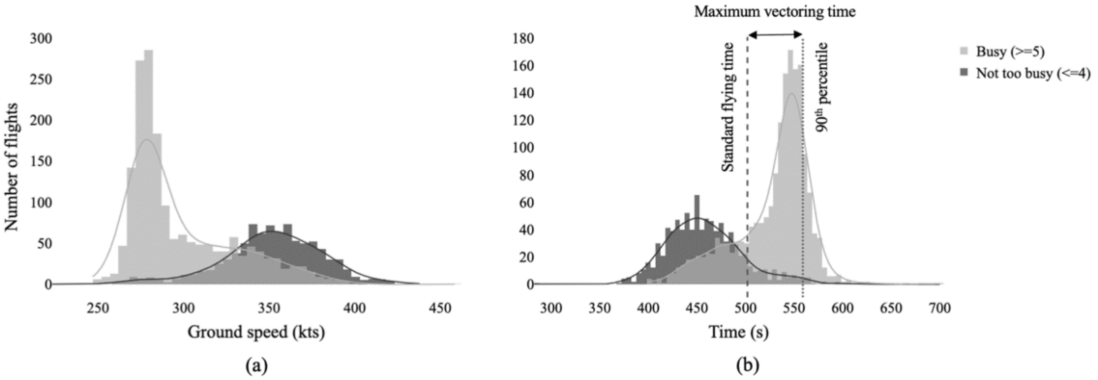
[1] Available online: https://www.ais.gov.hk/eaip_20220714/2022-07-14-000000/html/index-en-US.html



(a) (b)

Figure 2. Distribution of ground speed at MUSEL (a) and flying time in segment MUSEL-GUAVA (b).

Table 2. Parameters of segments in the simplified network.

Segment	Mean ground speed (kts)	Great Circle distance (km)	Standard travel time (s)	Maximum vectoring time (s)	Segment	Mean ground speed (kts)	Great Circle distance (km)	Standard travel time (s)	Maximum vectoring time (s)
ABBEY MUSEL	326 316	14.7	89	17	SIERA BORDA	372 310	36.0	205	32
MUSEL GUAVA	316 265	75.1	502	57	BORDA CANTO	310 324	31.2	191	391
BETTY MANGO	331 318	14.9	89	13	SIERA CANTO	372 324	40.6	183	42
MANGO GUAVA	318 265	76.8	512	55	CANTO MURRY	324 311	16.1	99	10
GUAVA LIMES	265 231	29.9	234	123	MURRY LIMES	311 231	39.0	280	153

3.3 Validation framework

To demonstrate the efficiency of the proposed MILP model, we design a framework for result validation, consisting of three principal components: historical data, an MILP model, and a flight simulation. In addition to tuning model parameters (as discussed in the Section *3.2.*), we extract entry flight information from historical data as the input source for the simulation. That said, arrival flights in the simulation are the digital mirrors of the ones coming to TMA in the past, with the same initial states when they were 50 km away from their entry waypoints. The second component, the MILP model, proposes optimal flight sequences based on the current situation every one minute. Meanwhile, the simulation, as its name indicates, stimulates aircraft movements and handles control tasks.

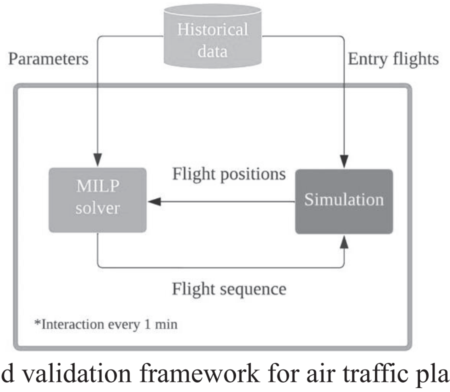


Figure 3. Proposed validation framework for air traffic planning algorithms.

4. RESULTS AND DISCUSSION

Based on the aforementioned framework, we generate a two-hour simulation for arrival flights in HKIA TMA from 18:00 to 20:00 HKT (during peak hours), on May 4th, 2018. The entire experiment is conducted on a single AMD Threadripper^{TM} 3990X core @ 2.9 GHz in 484 seconds. Therein, the MILP model is solved by Python-interface CPLEX (IBM, 2020) while the simulation is powered by equations and aircraft parameters from BADA (Nuic *et al.*, 2010). A demonstrating video is available at https://youtu.be/Vq62IG-sNQY.

Fig. 4 compares the average transit time (from 50 km prior to entry waypoints to LIMES) between historical and simulated flights coming from four different entry waypoints. Note that the initial conditions (including positions, heading angles, and speed values) of simulated flights are obtained from historical data. Upon completing the simulation, we notice that it gives a much more favorable state than historical data, with its airborne flights much nearer to LIMES. Therefore, it is better to consider only the middle flights (not in the initial and final states) in comparison. In general, aircraft in the simulation spend less time in TMA (with an average of 1108 seconds) than in the past (1218 seconds). Flights coming from CANTO experience a significant decrease in transit time while those from BETTY spend slightly more airborne time in the virtual environment.

Furthermore, the suggested MILP model provides fairer sequencing suggestions. The simulated results align well with the distance of each segment. In other words, the average transit time of flights from one entry waypoint is proportional to its distance to LIMES. In addition, for each entry waypoint group, statistics obtained from the simulation present a smaller variance than ones from history.

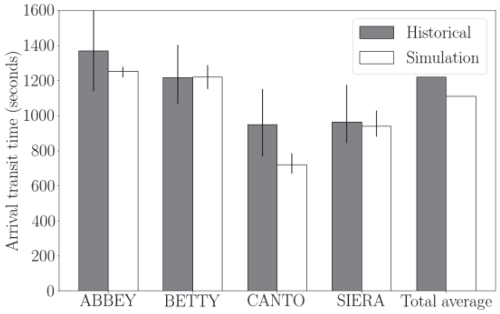


Figure 4. Comparison of arrival transit time between historical data and optimized sequences.

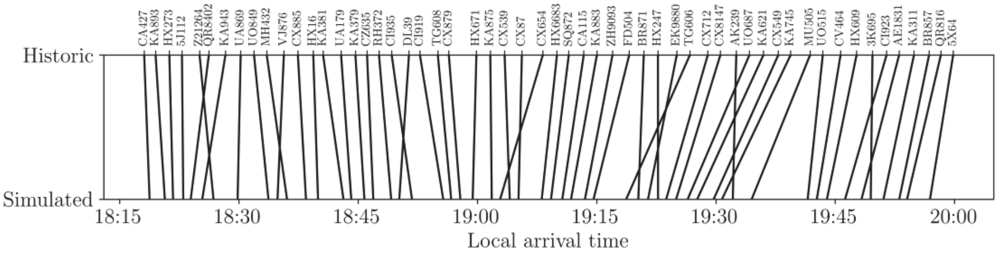


Figure 5. Arrival sequencing comparison between historical data and the simulation.

One may argue that the reduction of approximately 9% suggested by the MILP model is somewhat unrealistic due to the lack of weather consideration. In fact, weather conditions affect both the control capability of individual aircraft and separation constraints. As aircraft control is not the focus of this paper, we can only increase wake separation minima to mimic the network behavior in adverse weather conditions. Concretely, we add 0.5 nautical miles to wake separation minima of every pair of flights. The result this time remains high, with a more than 8% reduction in total transit time. This suggests that the proposed model truly recommends optimal landing sequences instead of nearly even spacing in historical data, as revealed in Fig. 5.

However, there exist several limitations in our current work. First, out of the research scope, control tasks such as holding, and vectoring are simplified. That may affect unfavorable subsequent states of the simulated environment since the aircraft are not able to move with more flexibility. Besides, in the simulation, aircraft are required to pass all waypoints, which is less efficient than in reality where ATC can let aircraft fly around waypoints to keep minimum separation with adjacent flights. Additionally, we will incorporate the influence of weather conditions in the simulated environment in an effort to enhance its authenticity.

5. SUMMARY REMARKS AND FUTURE WORK

In this article, we propose an MILP model, extensible and integrable with other methods, for air traffic coordination problems. We specifically apply the model to HKIA TMA with tailored constraints and appropriate assumptions. Furthermore, we introduce a validation framework for operation tests of air traffic control algorithms. Coordination results obtained through the validation framework demonstrate the efficiency of the suggested MILP model, reducing up to 9% of the total transit time.

Last but not least, with its simplicity yet comprehensiveness, this model is expected to play the role of a base layer on which other machine learning methods can be deployed with the purpose of automatic guidance in the future air traffic management system. Therefore, we will soon study such appropriate approaches for the top layer of the proposed model as well as the integrability between the two layers.

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